Exercise 15B: Solutions

1 Add the corresponding entries.

$$\mathbf{X} + \mathbf{Y} = \begin{bmatrix} 1+3 \\ -2+0 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Double each entry.

$$2\mathbf{X} = \begin{bmatrix} 2 \times 1 \\ 2 \times -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

Multiply each entry in $\mathbf{Y}$ by 4 and add the corresponding entry for $\mathbf{X}$.

$$4\mathbf{Y} + \mathbf{X} = \begin{bmatrix} 4 \times 3 + 1 \\ 4 \times 0 + -2 \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \end{bmatrix}$$

Subtract corresponding entries.

$$\mathbf{X} - \mathbf{Y} = \begin{bmatrix} 1-3 \\ -2-0 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

Multiply each entry by -3.

$$-3\mathbf{A} = \begin{bmatrix} -3 \times 1 & -3 \times -1 \\ -3 \times 2 & -3 \times 3 \end{bmatrix}$$
$$= \begin{bmatrix} -3 & 3 \\ -6 & -9 \end{bmatrix}$$

Add $\mathbf{B}$ to the previous answer.

$$-3\mathbf{A} + \mathbf{B} = \begin{bmatrix} -3 & 3 \\ -6 & -9 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 3 \\ -7 & -7 \end{bmatrix}$$

2

$$2\mathbf{A} = \begin{bmatrix} 2 & -2 \\ 0 & 4 \end{bmatrix}$$

$$-3\mathbf{A} = \begin{bmatrix} -3 & 3 \\ 0 & -6 \end{bmatrix}$$

$$-6\mathbf{A} = \begin{bmatrix} -6 & 6 \\ 0 & -12 \end{bmatrix}$$

3 a As the matrices have the same dimensions, corresponding terms can be added. They will simply be added in the opposite order.
Since the commutative law holds true for numbers, all corresponding entries in the added matrices terms will be equal, so the matrices will be equal.

b As the matrices have the same dimensions, corresponding terms can be added. The first matrix will add the first two numbers, then the third, and the second matrix will add the second and third numbers frst, then add the result to the first number.
Since the associative law holds true for numbers, all corresponding entries in the added matrices terms will be equal, so the matrices will be equal.

4 a Multiply each entry by 2.

$$2\mathbf{A} = \begin{bmatrix} 6 & 4 \\ -4 & -4 \end{bmatrix}$$

b Multiply each entry by 3.

$$3\mathbf{B} = \begin{bmatrix} 0 & -9 \\ 12 & 3 \end{bmatrix}$$

c Add answers to **a** and **b**.

$$2\mathbf{A}+3\mathbf{B}=\begin{bmatrix}6&4\\-4&-4\end{bmatrix}+\begin{bmatrix}0&-9\\12&3\end{bmatrix}$$
$$=\begin{bmatrix}6&-5\\8&-1\end{bmatrix}$$

d Subtract **a** from **b**.

$$3\mathbf{B}-2\mathbf{A}=\begin{bmatrix}0&-9\\12&3\end{bmatrix}-\begin{bmatrix}6&4\\-4&-4\end{bmatrix}$$
$$=\begin{bmatrix}-6&-13\\16&7\end{bmatrix}$$

5 a Add corresponding entries.

$$\begin{bmatrix}1&0\\0&3\end{bmatrix}+\begin{bmatrix}-1&1\\2&0\end{bmatrix}=\begin{bmatrix}0&1\\2&3\end{bmatrix}$$

b Triple entries in $\mathbf{Q}$, then add to corresponding entries in $\mathbf{P}$.

$$\begin{bmatrix}1&0\\0&3\end{bmatrix}+\begin{bmatrix}-3&3\\6&0\end{bmatrix}=\begin{bmatrix}-2&3\\6&3\end{bmatrix}$$

c Double entries in $\mathbf{P}$, then subtract $\mathbf{Q}$ and add $\mathbf{R}$.

$$\begin{bmatrix}2&0\\0&6\end{bmatrix}-\begin{bmatrix}-1&1\\2&0\end{bmatrix}+\begin{bmatrix}0&4\\1&1\end{bmatrix}$$
$$=\begin{bmatrix}3&3\\-1&7\end{bmatrix}$$

6 a If $2\mathbf{A}-3\mathbf{X}=\mathbf{B}$, then $2\mathbf{A}-\mathbf{B}=3\mathbf{X}$

$$3\mathbf{X}=2\mathbf{A}-\mathbf{B}$$
$$\mathbf{X}=\frac{2}{3}\mathbf{A}-\frac{1}{3}\mathbf{B}$$
$$=\frac{2}{3}\begin{bmatrix}3&1\\-1&4\end{bmatrix}-\frac{1}{3}\begin{bmatrix}0&-10\\-2&17\end{bmatrix}$$
$$=\begin{bmatrix}\frac{2}{3}\times 3-\frac{1}{3}\times 0&\frac{2}{3}\times 1-\frac{1}{3}\times -10\\\frac{2}{3}\times -1-\frac{1}{3}\times 2&\frac{2}{3}\times 4-\frac{1}{3}\times -17\end{bmatrix}$$
$$=\begin{bmatrix}2&4\\0&-3\end{bmatrix}$$

b If $3\mathbf{A}+2\mathbf{Y}=2\mathbf{B}$ then $2\mathbf{Y}=2\mathbf{B}-3\mathbf{A}$

$$\mathbf{Y}=\mathbf{B}-1\frac{1}{2}\mathbf{A}$$
$$=\begin{bmatrix}0&-10\\-2&17\end{bmatrix}-1\frac{1}{2}\begin{bmatrix}3&1\\-1&4\end{bmatrix}$$
$$=\begin{bmatrix}0-\frac{3}{2}\times 3&-10-\frac{3}{2}\times 1\\-2-\frac{3}{2}\times -1&17-\frac{3}{2}\times 4\end{bmatrix}$$
$$=\begin{bmatrix}-\frac{9}{2}&-\frac{23}{2}\\-\frac{1}{2}&11\end{bmatrix}$$

7 $\mathbf{X} + \mathbf{Y}$

$$= \begin{bmatrix} 150+160 & 90+90 & 100+120 & 50+40 \\ 100+100 & 0+0 & 75+50 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 310 & 180 & 220 & 90 \\ 200 & 0 & 125 & 0 \end{bmatrix}$$

The matrix represents the total production at two factories in two successive weeks.